WEBB INSTITUTE OF NAVAL ARCHITECTURE

EXPERIMENTAL DETERMINATION OF WAVE RESISTANCE OF A SHIP MODEL FROM LATERAL WAVE-SLOPE MEASUREMENTS

by

Lawrence W. Ward

This research was carried out under the Naval Ship Systems Command, General Hydromechanics Research Program administered by the Naval Ship Research and Development Center. Prepared under the Office of Naval Research Contract Nonr-4152 (00).

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Webb Institute of Naval Architecture
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ABSTRACT

A new method of determining ship model wave resistance from the wave pattern generated during a test in a model tank is outlined. This method is based on a Fourier analysis of lateral wave-slope data taken on a longitudinal cut parallel to the model path. Results of an exploratory test series carried out at Webb Institute are given and compared with previous results using a different method. The comparison is very encouraging and indicates that a new technique has been established which has many beneficial features, including potential application in full-scale research. Details of an electrical wave-slope measuring probe for use in the model tank are included in an appendix.

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NOMENCLATURE

Letters and Symbols

A, B, C, D	constants, functions, or points
A	wave amplitude, ft.
B	model beam, ft.
b	breadth of model tank, ft.
C	Fourier Cosine-transform; also resistance coefficient = $R/\frac{\rho}{2}SV_m^2$
C_b, C_p	block and prismatic coefficients
e	distance of cylinder and model off tank centerline, ft.
E	energy in wave system, ft. lbs.
F_r	Froude Number $= V_m/\sqrt{gL}$
g	acceleration of gravity, ft./sec.^{2}
H	model draft, ft.
h	depth of water in model tank, ft.
k	wave number, ft.$^{-1}$
K	constant
L	model length, ft.
p	pressure, lbs./ft.^{2}
R	resistance, lb.
s	element of length along path AB, ft.

S	Fourier Sine-transform; also wetted surface, $ft.^2$
u, v, w	fluid velocity components in the x, y, z directions, ft./sec.
u_ν , w_ν	Eggers' parameters, see Equation (15)
V_m	model speed, ft./sec.
V_n	fluid velocity normal to AB, ft./sec.
W	model weight, lb.
$\dot{W}$	energy transfer into wave system from rear plane, ft. lbs./sec.
x, y, z	Cartesian co-ordinates, ft. (see Figure 2) is vertically upwards.
α	angle of path with the x-axis, radians
γ_o	speed constant $gL/V_m^{\ 2} = 1/F_r^{\ 2}$
Δ	displacement, lbs.
ζ	wave elevation, ft.
ζ_y	lateral wave slope $= \partial\zeta/\partial y$, radians
θ	wave direction angle
λ	wave length, ft.
ν	index = 1, 2, ... ν_{max}
ρ	density of fluid, $slugs/ft.^3$
ϕ	velocity potential, such that $u = -\partial\phi/\partial x$, etc., $ft.^2/sec.$
ω	wave coordinate = $x \cos\theta + y \sin\theta$, ft.
Ω	ohms electrical resistance

Subscripts

o	centerline wave properties
y	lateral slope
I.T.T.C.	International Towing Tank Conference
t, f, r, w	total, frictional, residual, and wave parts

Superscripts, etc. (Illustrated for "Q")

$\dot{Q}$	time rate of change of Q
Q'	Q per unit distance in the x direction. Primes are also used to denote variations in a quantity.
$\bar{Q}$	average value of Q (see footnote page 6)

INTRODUCTION

In recent years several methods have been developed to determine ship model wave resistance experimentally in the model tank from measurements of the wave system generated by the model during a test. Interest in this subject was stimulated by a conference at Ann Arbor[1*] sponsored by the Office of Naval Research. The methods, several of which have been found to be successful, stem from essentially the same assumptions of linearized potential wave theory, but differ in the type of wave characteristic measured (e.g. wave elevation, slope, or force on a fixed object) and the type of path along which data are taken (e.g. a longitudinal or transverse cut, relative to the model path). Most depend upon a Fourier analysis of the data and yield the spectrum of the wave pattern as well as the wave resistance as the result. It should be noted that it is not necessary to assume the validity of any ship-wave generation theory (e.g. thin ship theory) for the derivation of these methods. An up-to-date picture of this work including a complete bibliography of papers related to this subject is contained in Reference (2).

The work done by the author at Webb Institute over the past few years[3,4] has primarily concerned his so-called "X-Y Method" which uses the x and y components of the lateral force exerted by the wave system on a long fixed vertical circular cylinder, and which does not require a Fourier analysis but yields the wave resistance directly. A major advantage of this scheme, as compared to those using wave elevation data, was found to be the finite length or "self truncation" of the record. This results from the rapid decay of the y force signal after passage of the model, before wave-reflection from the side walls of the tank becomes important. Recently a method with a similar

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*Raised numbers denote References listed beginning on page 40.

advantage has been tried utilizing lateral wave-slope data taken along a longitudinal cut. This method was first suggested along with a number of others by Sharma[5] in 1963 and yields the wave-slope spectrum as well as the wave resistance. Tests run at Webb Institute in June 1965, just before the author left for a one year National Science Foundation Fellowship in Hamburg, Germany, have now been analyzed, and this method has been found to be quite convenient and successful in the smaller facility at Webb. An attempt to include this method among others during a test series in the Hamburg model tank[2] did not meet with similar success, mainly because of the difficulties encountered there in connection with the slope probe instrumentation problem. However, in connection with the latter effort computational and theoretical evidence was found[2] which indicated this method to be consistent with some of the others which were used in the actual tests. This paper will outline the lateral wave-slope method and underlying theory briefly and present the test results and experience gained to date in its use.

THEORY

The basic underlying theory of wave survey methods in general can be found in several references, the latest being that by Eggers, Sharma, and Ward[2]. In this section a brief outline will be given of the theory needed for the lateral wave-slope method in particular, with emphasis on understanding the significance of the various steps involved in carrying it out.

Momentum Analysis

We consider the situation shown in Figure 1* of a model traveling at constant speed V_m along the centerline of a long tank of breadth b and depth h, and a set of axes x, y, z moving with the model. We consider potential flow and small surface disturbances, $\zeta(x,y,t)$. Consider a volumetric region I, fixed in space, whose side boundaries are formed by the lines ABCDC'B' projected down from the free surface except in the region of the model sides (where it is formed by them) and whose bottom surface is the tank bottom, both of the latter within the lines ABCDC'B'. We consider path with symmetry about the tank centerline, and therefore only the region ABCD on one side of the tank need be considered and the results doubled. The line CD is drawn sufficiently far ahead of the model so that there is no disturbance there. The line AB in general forms a curve as shown. The model moves ahead at the speed V_m and creates waves within region I. This wave system is often idealized to the concept of a transverse system 1 and a divergent system 2 contained within the cusp lines as

*Figures are numbered in order and located at appropriate places in the text. An index of Figures and Tables is given on page iii.

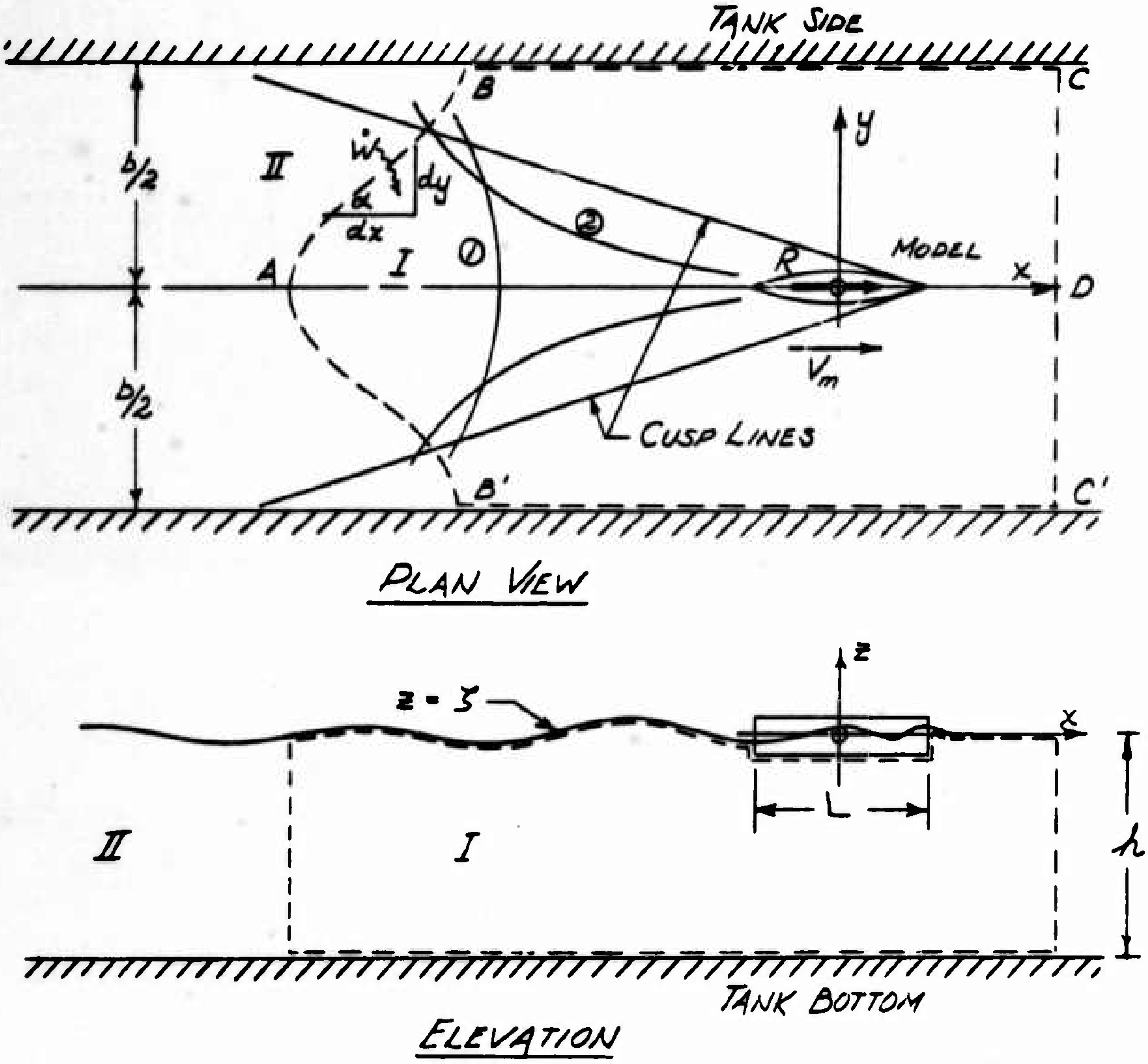

FIGURE-1 : ENERGY BALANCE CONTROL VOLUME

shown in Figure 1 and is often referred to as the Kelvin wave pattern. The wave system grows within this region and therefore the energy in the wave system increases at the rate:

$$\dot{E} = E' V_m \tag{1}$$

where E' is twice the average total wave energy in a strip of unit width in the x direction along the line AB extended down through the depth h. This growth of energy must be supplied by the work done at the boundaries of the region, this rate of doing work being given by integrals over the boundaries of the fluid of the pressure p times the velocity, V_n, normal to the boundary. There are no contributions to W from the tank sides BC and B'C' or the front plane CDC' or the tank bottom due to the normal velocity being zero in each case, nor from the free surface, due to the pressure there being zero ambient. Along the model sides there is work being put in, due to the action of the pressures there in conjunction with the forward motion of the model, at the rate $R_w V_m$, where R_w is the wave resistance, which is the only resistance possible in the present case in which an ideal fluid and small disturbances are assumed. Along the prismatic surface AB we have the work done by the waves in region II on region I, i.e. the "work done on the rear plane":

$$\dot{W}_{AB} = \int_{AB} ds \int_{-h}^{\zeta} p \, V_n \, dz \tag{2}$$

where p is the pressure, V_n is the fluid velocity normal to the surface AB, and ds is the element of distance along AB. Due to symmetry we

have $\dot{W}_{B'A} = \dot{W}_{AB}$. For the general case shown where the curve AB makes an angle ($\pi/2 - \alpha$) locally with the x axis we have:

$$V_n = U\frac{dy}{ds} - V\frac{dx}{ds} \tag{3}$$

where: $\frac{dx}{dy} = \tan\alpha$ and,

u and w are the fluid velocities in the x and y directions. The pressure p is given by the linearized Bernoulli relationship:

$$p = P_o + \rho U V_m - \rho g z \tag{4}$$

where terms of the order of the fluid velocities squared have been neglected in relationship to the unsteady pressure term $\rho U V_m$. Since the value of $\dot{W}$ for use in the energy relationship must be an average value with respect to time* at a fixed plane, the pressure times velocity terms represented by the first and third terms in Equation (6) will average to zero. Thus we have:

$$\dot{\overline{W}}_{AB} = 2\rho V_m \int_{AB} dy \int_{-h}^{\zeta} U(U - V\tan\alpha)\, dz \tag{5}$$

Since this is the only work term other than that due to the towing force we have, equating the total work put into region I to the average rate of growth of energy in the wave system in this region:

$$\left.\begin{aligned} \dot{\overline{W}}_{AB} + R_W V_M &= \overline{E}' V_m \\ \text{or} \quad R_W &= \overline{E}' - \dot{\overline{W}}_{AB}/V_m \end{aligned}\right\} \tag{6}$$

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* The time average $\bar{Q}$ of a quantity Q is defined as:

$$\bar{Q} = \frac{1}{\Delta t}\int_{t_o}^{t_o+\Delta t} Q(t)\, dt$$

where Δt is the period of fluctuation of Q or the limit can be taken as $\Delta t \rightarrow \infty$

The total energy E' is equal to the sum of the potential and kinetic energies per unit distance in the x direction:

$$\bar{E}' = \rho g \int_{AB} \zeta \, dy + \rho \int_{AB} dy \int_{-h}^{\zeta} (u^2 + v^2 + w^2) \, dz \quad (7)$$

where $z = \zeta(x,y)$ is the wave elevation and w is the fluid velocity in the z direction. Combining Equations (5), (6), and (7) gives the "1st Havelock Formula":

$$R_w = \rho g \int_{AB} \zeta^2 dy + \rho \int_{AB} dy \int_{-h}^{\zeta} (u^2 + v^2 + w^2) \, dz$$
$$- 2\rho \int_{AB} dy \int u(u - v \tan\alpha) \, dz \quad (8a)$$

for the general path AB. Two special cases are of interest: For plane perpendicular to the direction of motion we set $\alpha = 0$, giving:

$$R_w = \rho g \int_0^{b/2} \zeta^2 dy + \rho \int_0^{b/2} dy \int_{-h}^{\zeta} (-u^2 + v^2 + w^2) \, dz \quad (8b)$$

This is the form used for a transverse cut method. For a longitudinal cut method we set $\alpha = \pi/2$, giving:

$$R_w = 2\rho \int_{-\infty}^{\infty} dx \int_{-h}^{\zeta} uv \, dz \quad (8c)$$

Fourier Wave-Spectrum Model

A study of Equation (8a) will convince the reader that this relationship does not form a convenient basis for an experimental method, as it requires determination not only of the wave elevation along the path AB but of the fluid velocities u, v, w throughout the depth of the fluid along the vertical plane formed by the line AB extended in depth into the fluid. We wish to use potential wave theory, assuming free gravity waves of small amplitude, in effect to predict these fluid velocities from knowledge of the wave elevation (or in the present case from knowledge of the wave slope in the y direction $\zeta_y = \partial\zeta/\partial y$). This will allow us to "trade-off" more detailed knowledge of the function ζ on the surface for the next-to-impossible integration in depth. This would be quite simple for a single component wave of amplitude A and length λ traveling in a direction θ to the x axis:

$$\left.\begin{aligned} \zeta &= A\cos k\omega \\ \text{where:}\quad \omega &= x\cos\theta + y\sin\theta \\ k &= 2\pi/\lambda \end{aligned}\right\} \qquad (9)$$

Potential gravity wave theory would then predict:

$$\left.\begin{aligned} U &= -\partial\phi/\partial x \\ V &= -\partial\phi/\partial y \\ W &= -\partial\phi/\partial z \\ \text{where:}\quad \phi &= \frac{Ag}{kc}e^{kz}\cos k\omega \end{aligned}\right\} \qquad (10)$$

and $c = \sqrt{\dfrac{g\lambda}{2\pi}}$ is the wave speed in deep water which is assumed. The

wave system which is likely to be generated by even a simple traveling disturbance is quite complex, however[6]. For this reason we must use a spectrum representation:

$$\zeta = \int_{-\pi/2}^{\pi/2} \left[A(\theta) \cos k\omega + B(\theta) \sin k\omega \right] d\theta \qquad (11)$$

This can be seen to be the superposition of a large number of waves of amplitude $A(\theta)d\theta$ traveling in different directions with a varying phase relationship allowed by $B(\theta)$. A similar spectrum form exists for ϕ and therefore the velocities u, v, w are known in terms of the spectrum functions A and B. Moreover simple differentiation would result in the ζ_y or "lateral wave-slope" spectrum:

$$\zeta_y = \int_{-\pi/2}^{\pi/2} \left[A_y(\theta) \cos k\omega + B_y(\theta) \sin k\omega \right] d\theta \qquad (12)$$

where $A_y(\theta) = k \sin\theta \, B(\theta)$, etc.

It can therefore be seen that knowledge of the spectrum of one characteristic of the wave system (say ζ_y) will in theory allow any other to be derived analytically. This is an important point which will allow one to pick the most convenient data to measure experimentally, regardless of which type of spectrum is needed. Some typical wave spectra can be seen in Figures 5(a)-(e), and further discussion on the matter of wave patterns can be found in Reference 6.

Stationary Wave-Pattern Requirement

Another very important fact about the ship wave system is that it must be stationary with relation to the ship. The corresponding relationship between k and θ, which can be derived either geometrically or from mathematical reasoning[7], is:

$$k = k_o \sec^2\theta \tag{13}$$

where: $k_o = g/V_m^2$

Other useful relationships are:

$$\left.\begin{aligned} \lambda &= \lambda_o \cos^2\theta \\ c &= c_o \cos\theta \end{aligned}\right\} \tag{14}$$

where c is the wave speed, and the zero subscript denotes the wave component for $\theta = 0$, or the "centerline" wave. It should be noted that all other waves are shorter and slower than the centerline wave, and that $c_0 = V_m$.

Eggers w_ν, u_ν Parameters

Using the foregoing relationships the argument ($k\omega$) of

the wave functions can be written:

$$\left.\begin{aligned} k\omega &= w_\nu x + u_\nu y \\ \text{where:} \quad w_\nu &= k_o \sec\theta \\ u_\nu &= k_o \sec\theta \tan\theta \end{aligned}\right\} \qquad (15)$$

The factors w_ν and u_ν are the convenient "wave numbers in the x and y directions" introduced by Eggers[8], and are used as parameters in the analysis to follow. For this purpose however they are non-dimensionalized as follows:

$$\left.\begin{aligned} w_\nu' &= w_\nu / k_o \\ u_\nu' &= u_\nu / k_o \end{aligned}\right\} \qquad (16)$$

This form will be used with the primes not written in this text. It is seen that now there is only one parameter in the spectrum representation; either θ, w_ν or u_ν with the other two derived from it. Many different forms of the previous equations and those to follow are therefore possible.

Wave Resistance Formula

With the wave spectrum model now established, the 1st Havelock Forumla (8) can now be converted to integrations involving only the function ζ, that is the surface wave elevation. The resulting equations will appear either (a) in terms of the wave spectrum functions A and B or (b) in terms of the related Fourier transform coefficients (denoted S, C, etc.) of some type of wave data (ζ , ζ_y etc.) along some path (longitudinal or transverse cut, etc.), or (c), in the special case of the X-Y Method, in terms of a

direct integral of a ζ-function along a path. The latter is fully discussed in Reference (3). The first mentioned is the classical "2nd Havelock Formula"[9]:

$$R_w = \frac{\pi}{2}\rho V_m^2 \int_0^{\pi/2} (A^2+B^2)\cos^3\theta \, d\theta \quad , \tag{17}$$

which is extremely simple in form and requires only the magnitude of the wave spectrum $A(\theta)$, $B(\theta)$ to be known. The second-mentioned is very closely related to it and gives, for lateral-slope data taken on an infinite longitudinal cut[5]:

$$R_w = \frac{\rho V_m^2}{2\pi} \int_0^{\infty} (S_y^2 + C_y^2) \frac{dU_\nu}{W_\nu^4(2W_\nu^2-1)} \tag{18}$$

where:

$$\begin{Bmatrix} S_y \\ C_y \end{Bmatrix} = \int_{-\infty}^{\infty} \zeta_y \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} (W_\nu k_o x)\, dx \tag{19}$$

are the sine and cosine transforms of the lateral slope data along a line y = constant. The infinite limits are in practice replaced by a finite integration over the extent of the signal which for the lateral slope data is finite due to its self-truncation characteristics noted previously. It is interesting to note that the θ form of (19):

$$R_w = \frac{\rho V_m^2}{2\pi} \int_0^{\pi/2} (S_y^2 + C_y^2)\cos^3\theta \, d\theta \tag{20}$$

is very similar in form to (17). The Eggers' parameters u_ν and w_ν are more convenient to use however in the computations.

Choice of Path

It should be noted that the results R_w and S_y, C_y should not depend on the path chosen; however both theoretical reasoning and computational experience show that the line y= constant should be neither too near the model or the tank wall for accurate results. The latter is due to tank wall reflections, not considered in the foregoing, and the former due to the presence near the model of a "local" wave system for which the wave spectrum model as given in Equations (11) and (12) is not a valid representation. Further guidance on this question can be obtained in Reference (2). In the case of lateral slope data, for reasonable distances of the probe from the model, tank wall reflection in a tank of normal breadth does not occur until the record has truncated itself; an advantage which this method shares with the X-Y Method[3], and which is not in general true of other longitudinal cut methods[2]. If the tank breadth is a critical factor, some advantage can be gained by employing "off-centerline towing" as described in Reference (3) and shown in Figure 2. A longitudinal cut is accomplished most easily by employing a fixed measuring location over the water surface near the tank centerline and letting the model pass by.

In the foregoing, port-starboard symmetry of the wave system and therefore of the spectrum has been assumed and therefore only one path on one side of the model is necessary. If such symmetry cannot be assumed (e.g. a heeled-yacht test), two paths will be necessary in practice to cross the wave-pattern in the "Kelvin sense,"[6] thus requiring simultaneous recording from two probes.

EXPERIMENTS

Tests

Experiments were carried out on June 4, 1965 at Webb Institute on a 5'-0" model of the Series 60, 0.60 Block Coefficient form. Particulars of this hull form are given in Table I.

Table I: Characteristics of the Series 60 Model

(See Reference 10)

Length Between Perpendiculars	L	5.000 ft.
Waterline Length		5.084 ft.
Beam	B	0.667 ft.
Draft	H	0.267 ft.
Displacement	Δ	31.170 lbs. FW
Wetted Surface	S	4.262 ft.^{2}
Block Coefficient	C_b	0.600
Prismatic Coefficient	C_p	0.614
Displacement-Length Ratio	$\Delta / 2240 (L/100)^3$	122

Stimulators: 1/8" Diameter x .050" height pins spaced 0.275" placed 4" aft of bow

The model basin at Webb Institute is 93 ft. long with a 10 ft. wide by 5 ft. deep rectangular section and a carriage on an overhead rail. The model was towed at a distance e_1 = 6in. off the tank centerline with a slope measuring probe mounted at a distance e_2 = 6-7/8 in. off the tank centerline on a fixed arm at about the mid-length of the tank but on the opposite side, as shown in Figure 2. This maximizes the length

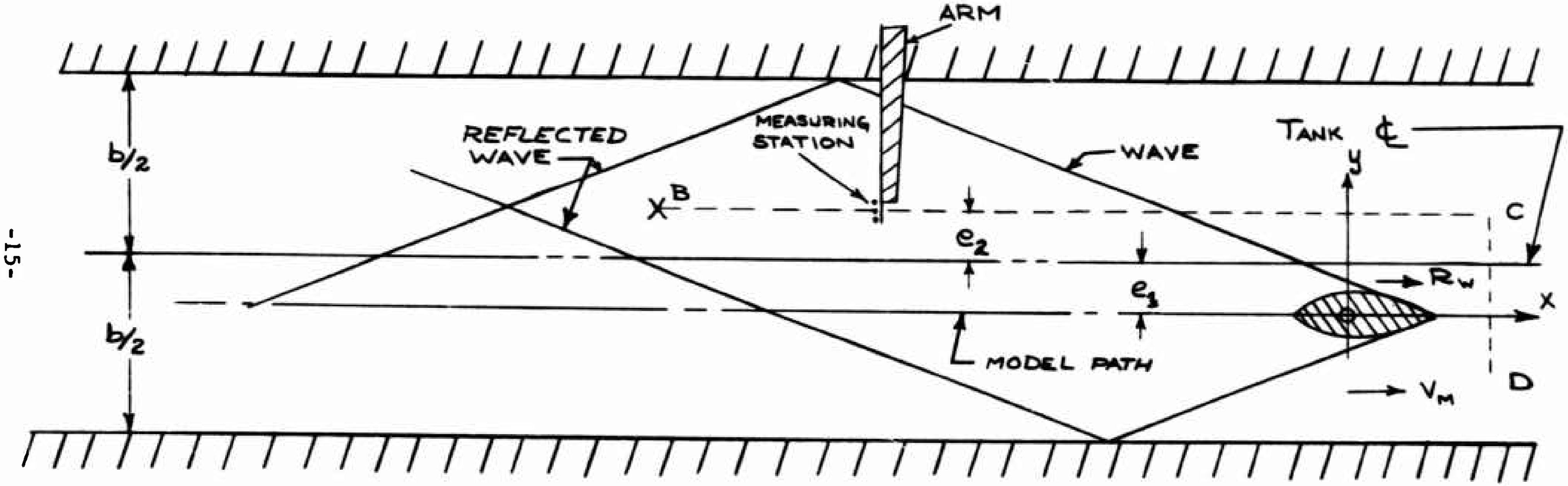


FIGURE 2: TEST GEOMETRY

of the resulting record along the line AB before wave reflection from the sides of the tank take place.

Records were taken of the lateral slope $\partial\zeta/\partial y$ of the waves caused by the model measured by a three-wire resistance type slope gage developed in conjunction with the work and described in detail in Appendix A. This instrument was designed to produce a linear reading, on a two channel Brush RD-4622-00 amplifier and Mark II recorder, proportional to the wave-slope in the y direction over a range of $\pm$ 0.20 radians and to ignore changes in the wave elevation over the maximum range encountered. Three repeat runs each were made at three different speeds over the range covered previously for this model in the "X-Y" Method" tests, in order to provide an exploratory series to examine the method itself. These conditions as well as the resulting resistance values are shown in Table II.

Table II: Test Results from the Lateral Slope Tests

Run on 6/4/65 on the Series 60, 0.60 Block Model

Run No.	Model Speed Vm, ft./sec.	Froude Number F_r	Effective Tank Width b_{eff}, ft.	No. Calc. Steps ν max	Resistance Coefft. 10^3 C_w
D-1	3.54	.279	12.2	51	.655
D-1	3.54	.279	12.2	76	.666 = +2%
D-2	3.51	.277	12.0	51	.[illegible]9
D-3	3.50	.276	12.0	51	.494
G-1	4.29	.338	18.0	51	1.576
G-2	4.30	.339	18.1	51	1.480
G-3	4.30	.339	18.1	51	1.533
J-1	5.09	.401	25.3	51	5.759
J-2	5.07	.400	25.1	51	5.772
J-3	5.09	.401	25.3	51	5.994

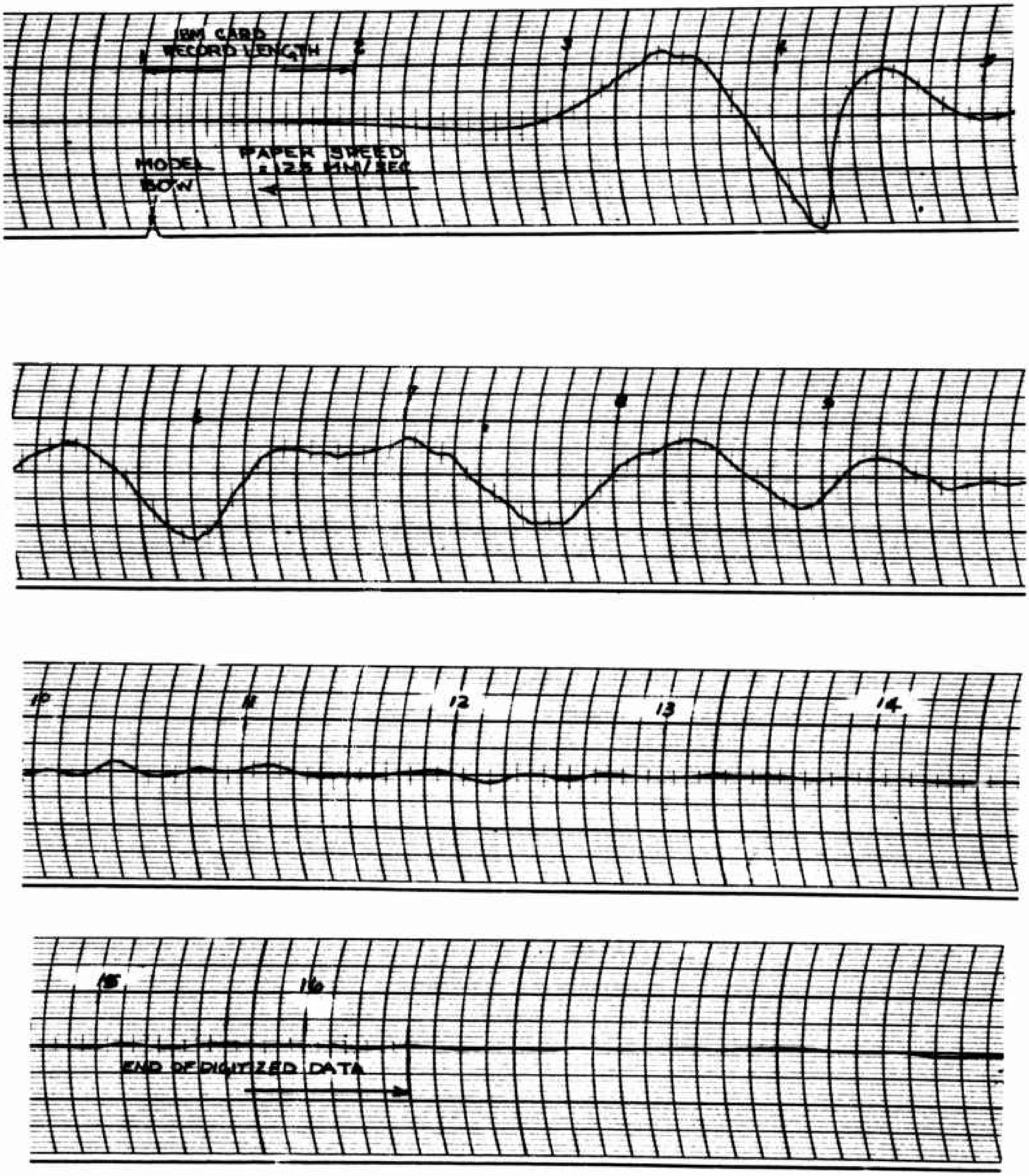
A sample record, Run G-2, is shown in Figure 3. It can be seen that this record is free from electrical or mechanical "noise". The event mark indicates passage of the model bow past the measuring station. The recorder was run at its maximum paper speed of 125mm/sec. to obtain an easily readable record. The calibration factor for the tests, converted to the sensitivity setting of this tape, was 85.0 mm radian, corresponding to a full scale tape reading of $\pm$ 0.17 radians. Less than 1.0 mm deflection was noted over a change of elevation of $\pm$ 1.0". As can be seen in Figure 3, the record was practically self-truncating before wave reflection from the tank walls took place. The tests were all accomplished, including set-up of equipment and calibrations before and after, in one working day.

Although turbulence stimulation is not as important in this type of testing as in the case of ordinary resistance tests, a turbulance stimulation pin configuration found suitable for this model in the high speed range was fitted as shown in Table I, however the tank water was not heated but was at the ambient temperature of approximately 72°F. The regular side cylindrical wave dampers and end corner beaches were in place during the tests. Speed measurement was accomplished by means of a timer acting between switches on the rail 35.0 ft. apart, and the model was towed by an arm on a fixed strut on the carriage.

Analysis

Prior to actual analysis, each tape (e.g.Fig. 3) was divided into 2.5 mm spacing (½ the divisions shown) and numbered and the data read off and transferred to IBM cards, thus providing the raw data in a usable "digitized" form. While this was of necessity done by hand, the instrumentation would be easily adapted to automatic digitizing by suitably designed tape

Figure 3:
Sample Record (G-2)



apparatus with a "playback" feature to realize the scanning rate of 50/sec. corresponding to the hand calculation.

The analysis itself consists of two steps. First is the carrying out of the two x integrations over each record, given by Equation (19), at a series of values of the "x-wave number" w_ν to give the Fourier sine and cosine transforms S_y and C_y as a function of w_ν . These are in themselves a result and can be plotted. The second step is the integration of the sum of the squares of these with the weighting function:

$$K(w_\nu) = \frac{1}{w_\nu^4 (2 w_\nu^2 - 1)} \tag{21}$$

in accordance with Equation (18), to give the wave resistance R_w for the tape analyzed at the speed V_m of the run. These results are non-dimensionalized in terms of the resistance coefficient C and the Froude number F_r given by:

$$\left.\begin{aligned} C &= R / \tfrac{\rho}{2} S V_m^2 \\ F_r &= V_m / \sqrt{g L} \end{aligned}\right\} \tag{22}$$

where S is the wetted surface and L the length of the model. The integrations are done by trapezoidal rule with steps of u_ν given by:

$$u_\nu = \frac{\pi \nu}{k_o b_{eff.}} \tag{23}$$

where b_{eff} is a hypothetical effective tank width corresponding to the exact Eggers series-spectrum model of the same spacing. In the present case a choice of:

$$k_o b_{eff} = 10 \pi \tag{24}$$

was made, thus giving steps Δu_ν of 0.10. The corresponding steps of w_ν are not even and are given by:

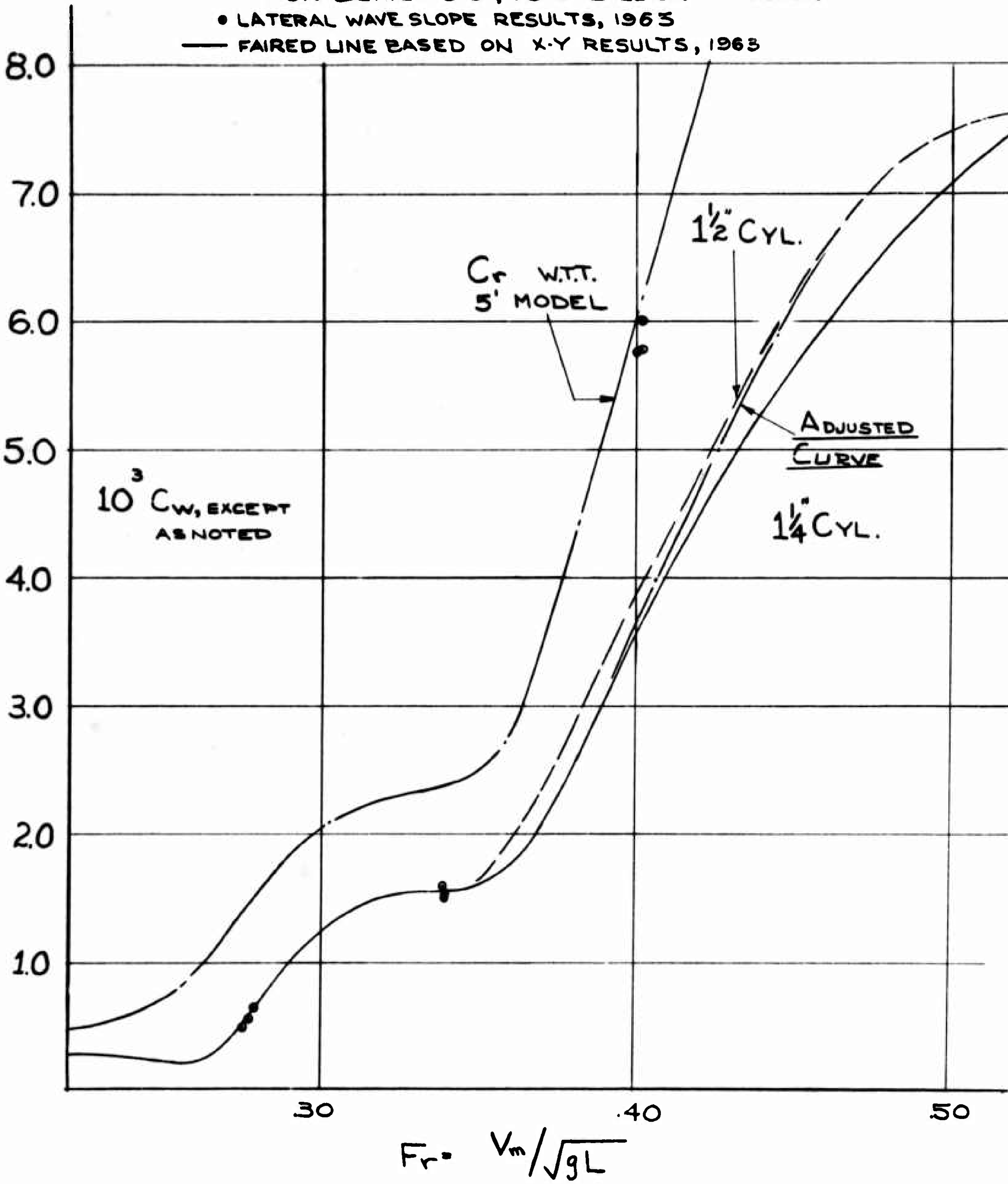
$$W_\nu^2 = \frac{1}{2}\left(1 + \sqrt{1 + 4U_\nu^2}\right) \tag{25}$$

The Fortran computer program designed to carry out the above calculation is described in Appendix B. A cut-off corresponding to $\nu_{max.} = 51$ was found to be sufficient in terms of no further contribution to the resistance values beyond this point, although the spectrum itself continues, because of the strong effect of the weighting function $K(w_\nu)$ in Equation (18); as shown in Table II extension to $\nu_{max.} = 76$ in the case of tape D-1 gave only a 2% increase in the result. The calculations were run on an IBM-1130 computer. Analysis time for the final set of calculations was about 3 minutes per ship run.

Results

The resulting wave resistance coefficients from this set of tests are given in Table II and compared with the previous results[3] using the X-Y Method, which are shown as a faired line, in Figure 4. Also included in this figure is the residual resistance coefficient C_r derived from a previous total resistance test of the same model at Webb. It can be seen that excellent agreement is present between the lateral slope results and the faired line at the lowest and middle speed tested, and that the lateral slope results are higher than the XY at the highest speed, but still less than the residual. This latter difference is considered to be in the right direction, as the XY results are known to be in error above

FIGURE 4: RESULTS OF LATERAL WAVE-SLOPE TESTS ON SERIES 60, .60 BLOCK MODEL



a Froude number of 0.34 for these tests due to the XY cylinder being of too small a diameter. Moreover, it can be argued that the measured wave resistance should approach the residual in the range of high Froude numbers. The basic validity of the X-Y Method has been further established by means of recent tests[2] in Hamburg, Germany on a larger model. Thus from the limited number of results obtained so far the lateral slope method appears to be a valid technique for measurement of wave resistance in the model tank, giving results as good as and perhaps better than the other methods used to date.

Several of the lateral wave slope spectrum results are shown in Figures 5(a)-(e). While these cannot be compared with any other established result, certain known features of such spectra can be examined. One is the presence of zeros in the ideal fluid case or near-zeros in the actual case of a real fluid of small viscosity. Physical reasoning shows[6] that these should be spaced, when plotted against a base of w_ν, approximately at an interval of:

$$\Delta w_\nu = \pi / \gamma_o \tag{26}$$

where $\gamma_o = k_o L/2$. It can be seen in Figure 5(b) for the case of G-2 that this is approximately true. The trend of movement of these "zeros" away from the axis as the speeds are increased might be the result of worsening separation at the higher speeds. In addition the consistency of the resulting spectra can be noted for the highest speed tested in Figures 5(c)-(e); this is not too bad considering the accuracy of measurement possible and the complexity of the analysis. An additional factor which might contribute to such variations is a port-starboard fluctuation of the wave system which has sometimes been noticed during model tests.

The weighting function $K(w_\nu)$ in Equation (11) is also graphed in Figure 5(b). From this we can see that only a portion of the spectrum,

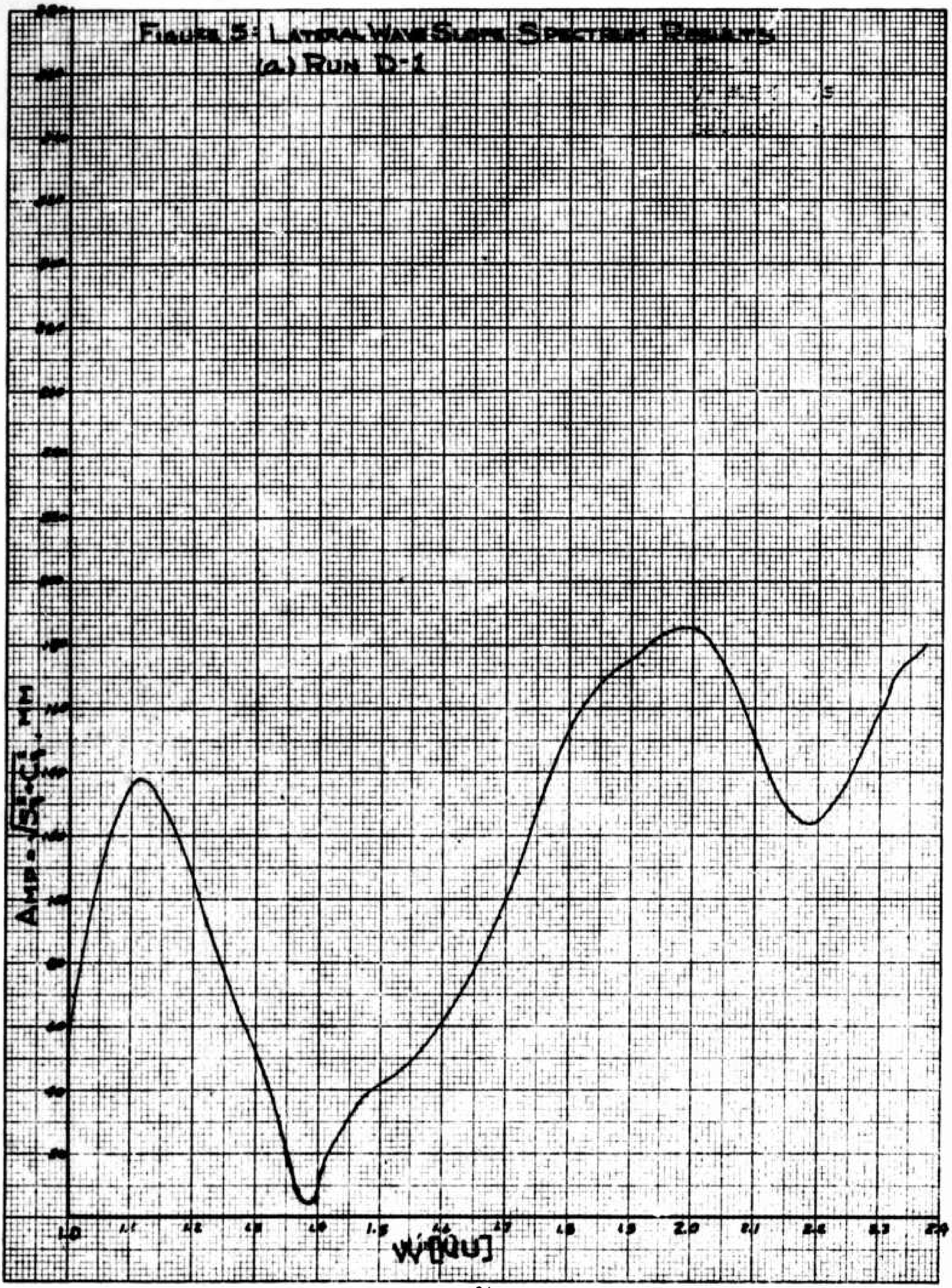


FIGURE 5: LATERAL WAVE SLOPE SPECTRUM RESULTS
(a) RUN D-1

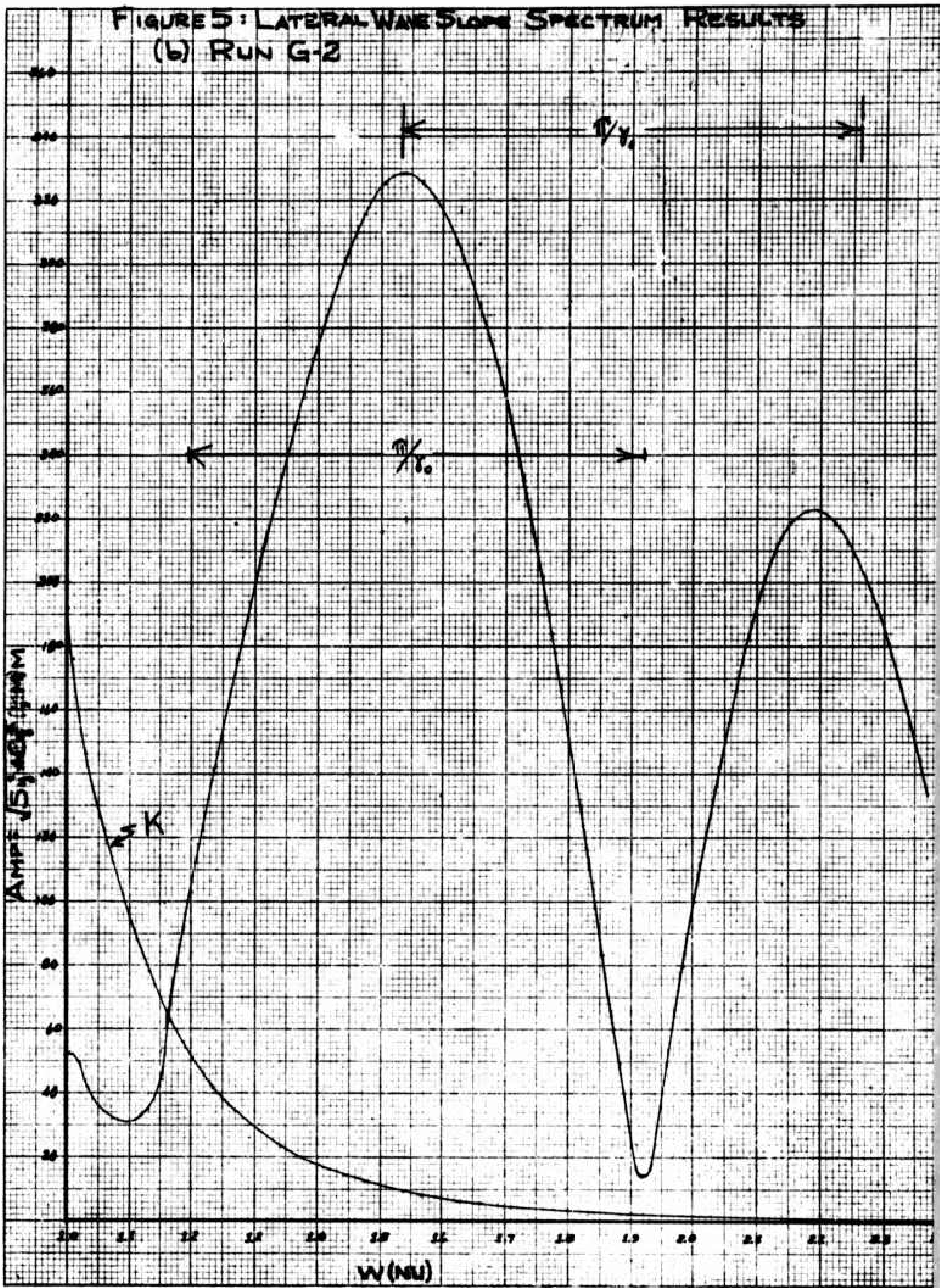


FIGURE 5: LATERAL WAVE SLOPE SPECTRUM RESULTS
(b) RUN G-2

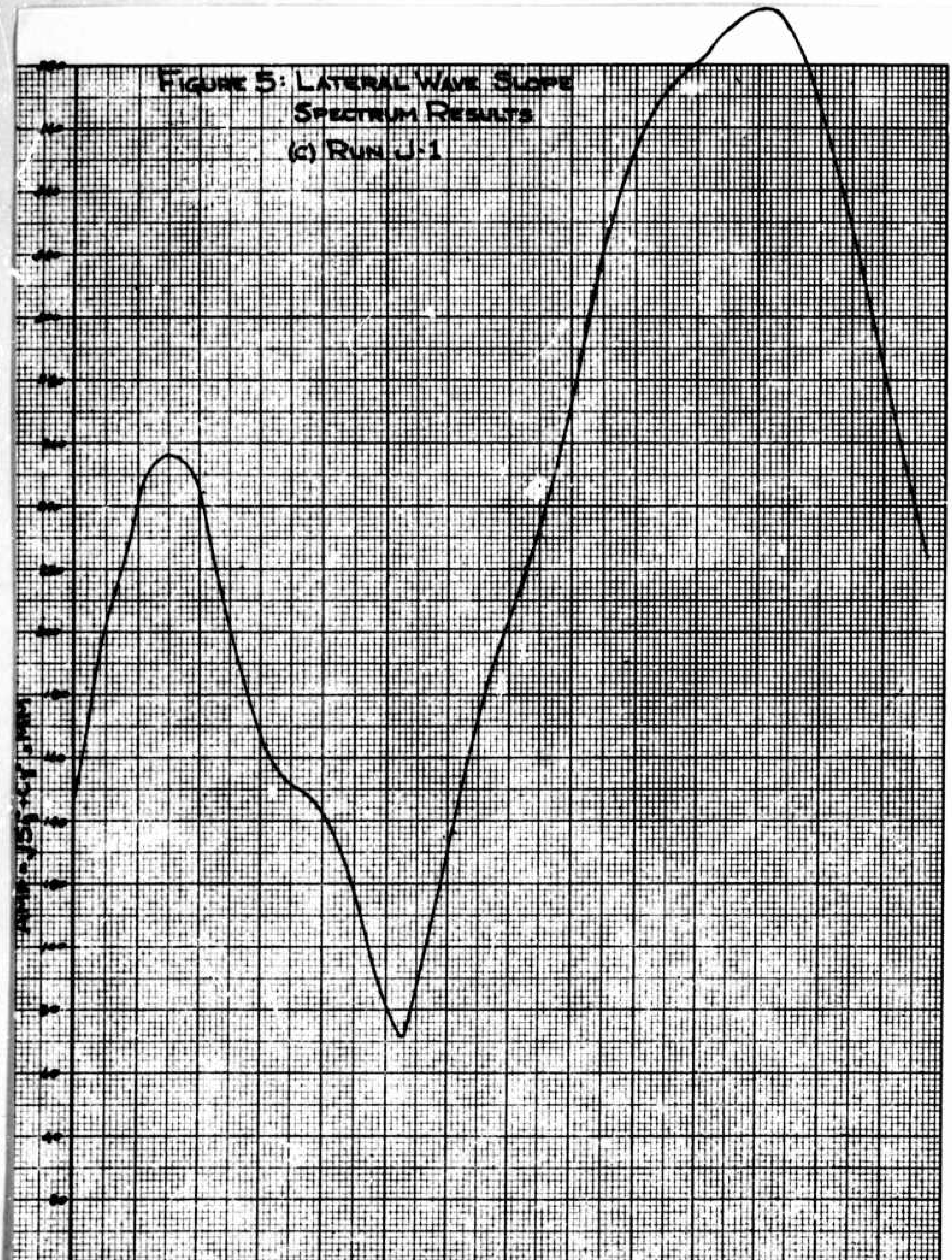


FIGURE 5: LATERAL WAVE SLOPE SPECTRUM RESULTS
(c) RUN J-1

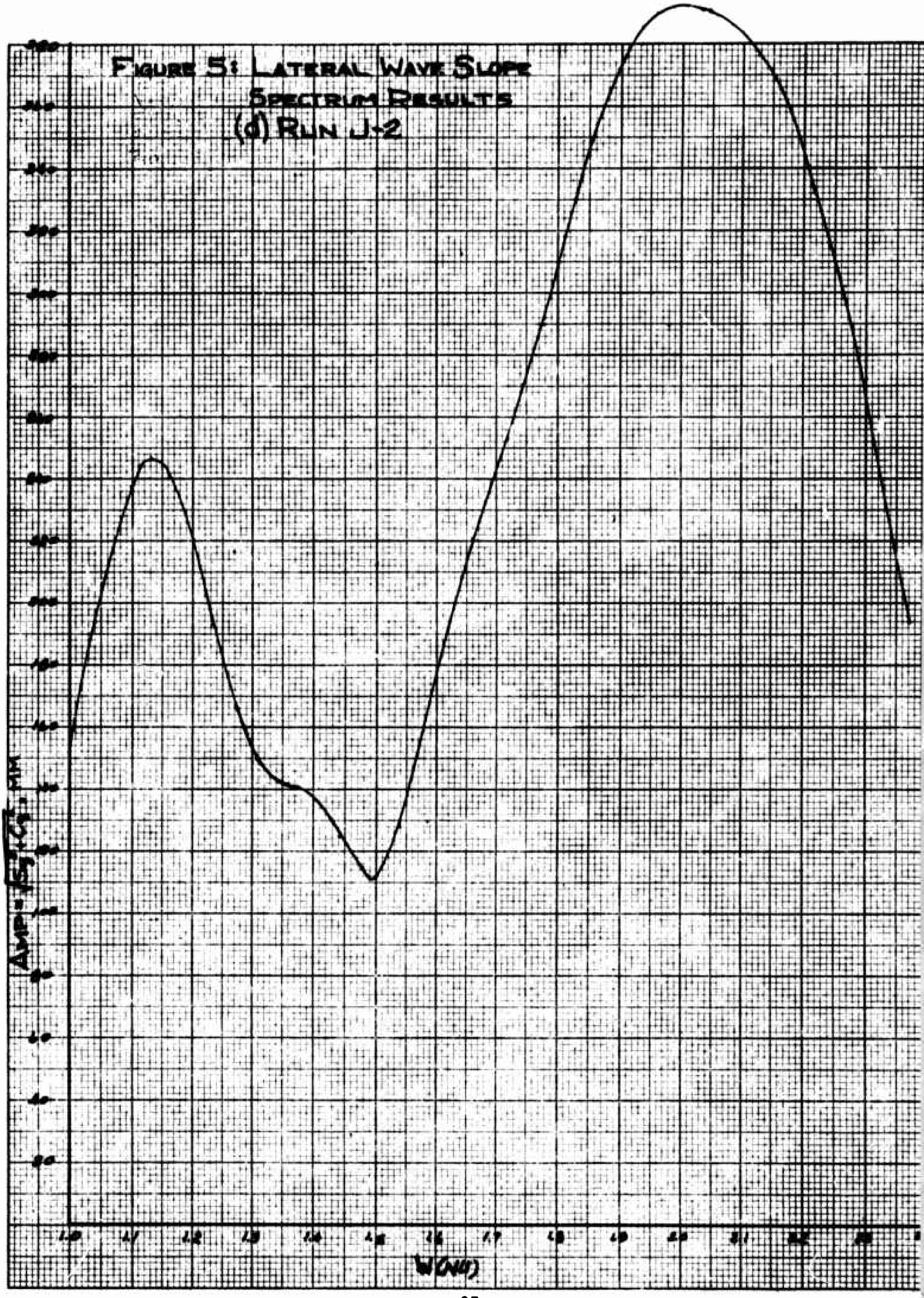


FIGURE 5: LATERAL WAVE SLOPE SPECTRUM RESULTS
(d) RUN J-2

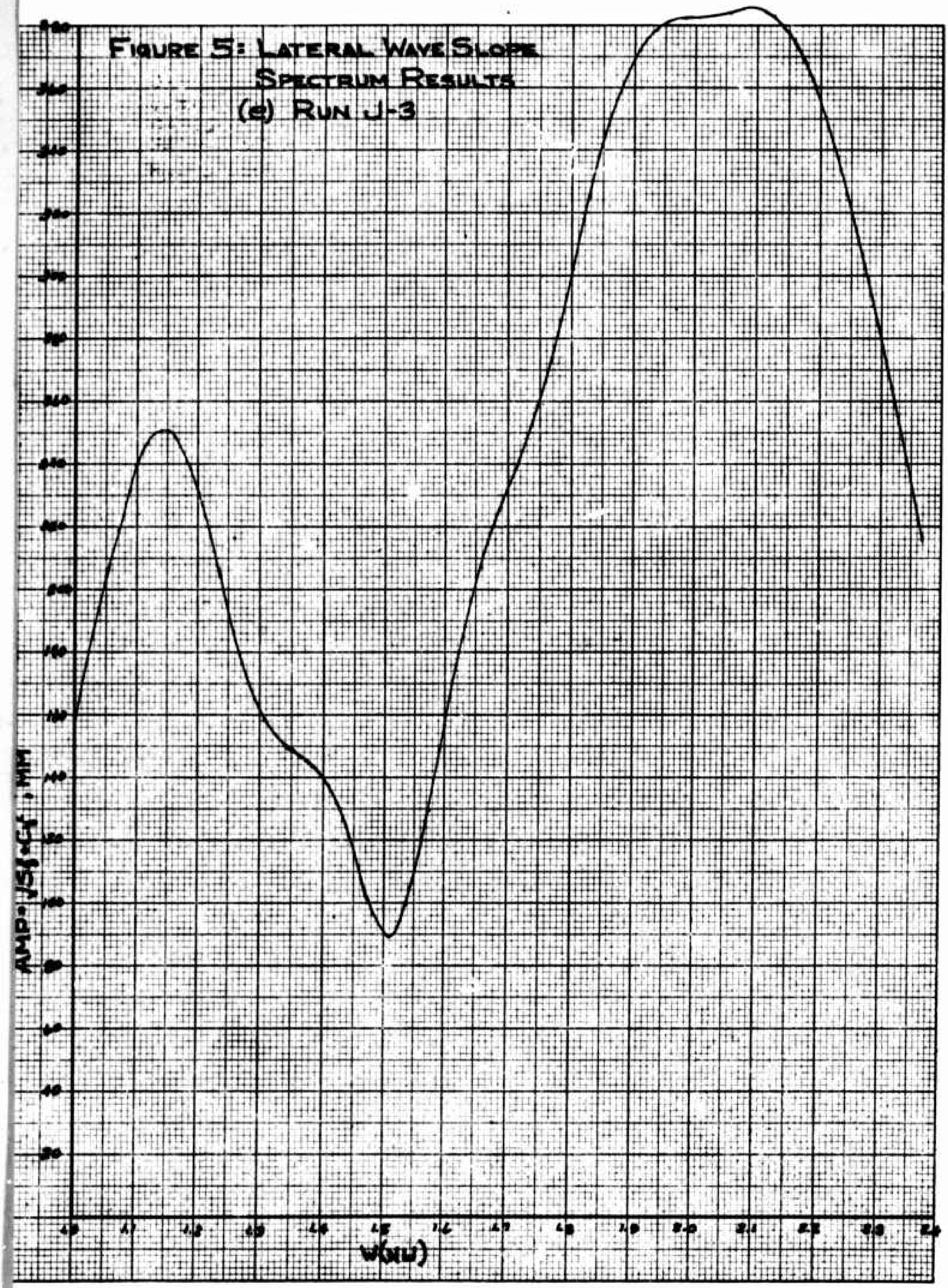


FIGURE 5: LATERAL WAVE SLOPE
SPECTRUM RESULTS
(e) RUN J-3
W(XIW)

that for $0<\theta<60°$ approximately, seems to be needed for the accurate determination of the wave resistance. This "paradox" is related to the fact that each part of the ship wave spectrum is analytically related to each other as discussed in Reference (6).

It should be mentioned that the above results were preceeded by an independent calculation by S. D. Sharma at the Hamburgische Schiffbau-Versuchsanstalt in Hamburg, Germany on the runs D-1 through D-3 which yielded results pratically identical with those exhibited here. An interesting feature in one of these results, which was carried out to a much larger range of w_{ν} than necessary to calculate the wave resistance, is the persisting nature of the oscillations in the spectrum at a constant interval close to the value π/γ_0.

CONCLUSIONS AND RECOMMENDATIONS

Based on comparison of the results of the exploratory tests described herein with other related results in the area of wave survey testing, it is concluded that the lateral wave-slope method is a valid experimental technique for determining the wave resistance and corresponding wave spectrum of a ship model in a typical model tank. Both the necessary instrumentation and the tests and analysis were accomplished with reasonable ease and accuracy in the relatively small facility at Webb Institute. Further investigations of the method and technique should include dynamic calibration of the slope probe as well as experiments at varying lateral positions of the measuring station. Experience with other longitudinal cut methods[12] has shown that the latter might have an influence on the results. In addition it would be useful to apply the method to other types of ship forms and other vehicle types to gain further experience with these and with the measuring problems involved.

It is considered that the method can now be applied to solving useful and practical problems for which it is ideally suited as well, such as optimizing bulbs and other hull form variations. Provision of this equipment can "stretch" the capability of a model tank, especially one in the smaller size range. Also, since this method has potential application in near or full-scale testing, some development should be considered in this direction. Such studies should include consideration of ambient sea noise as well as more practical matters such as wave-slope measuring buoys, etc.

Although the measurements and analysis in the present series are not considered burdensome, for more extensive use of this technique on a regular basis automatic digitizing including print or punch-out of the

recorded slope signal would be recommended. This would allow more runs to be made and more different conditions analyzed as well as producing almost immediate results after a test.

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APPENDIX A: DEVELOPMENT OF THE WAVE-SLOPE MEASURING PROBE

A successful wave-slope measuring probe based on the resistance wire principle was developed in 1965 for use at Webb in conjunction with the 2KC excitation from a two-channel Brush RD-4622-00 amplifier and Mark II recorder. The probe and the calibration scheme is shown in Figure A-1 and the corresponding circuit diagram is shown in Figure A-2. The scheme is an adaptation of the wave height measuring probe previously developed in connection with this effort[3], and employs three parallel stainless-steel wires of .004 inch diameter, spaced at 3/4 inch intervals between carefully drilled holes in Plexiglass blocks held by a C-shaped frame. The frame is mounted on a slider which allows known variations of vertical height of $\pm$ 3" at approximately ½" intervals and this in turn is on a plate which can be rotated in either direction to produce a slope variation of $\pm$ .20 radians in steps of 0.10 for the slope calibration. The height variations are used to calibrate wave height probes mounted on the same frame or in the present case to check the insensitivity of the wave-slope probe to changes in wave elevation.

As shown in Figure A-2, the measuring circuit consists of a Wheatstone bridge with two of the resistances R_1 and R_4 replaced by the equivalent path resistance of the water between each set of wires AB and AC. This scheme is known in the case of the height probe to give a good linear response to changes in wave height over a range of several inches when the ratio of resistance:

$$R_3/R_4 = R_2/R_1 = K \qquad \text{(A-1)}$$

is properly chosen; in this case the value of K is approximately 20 based

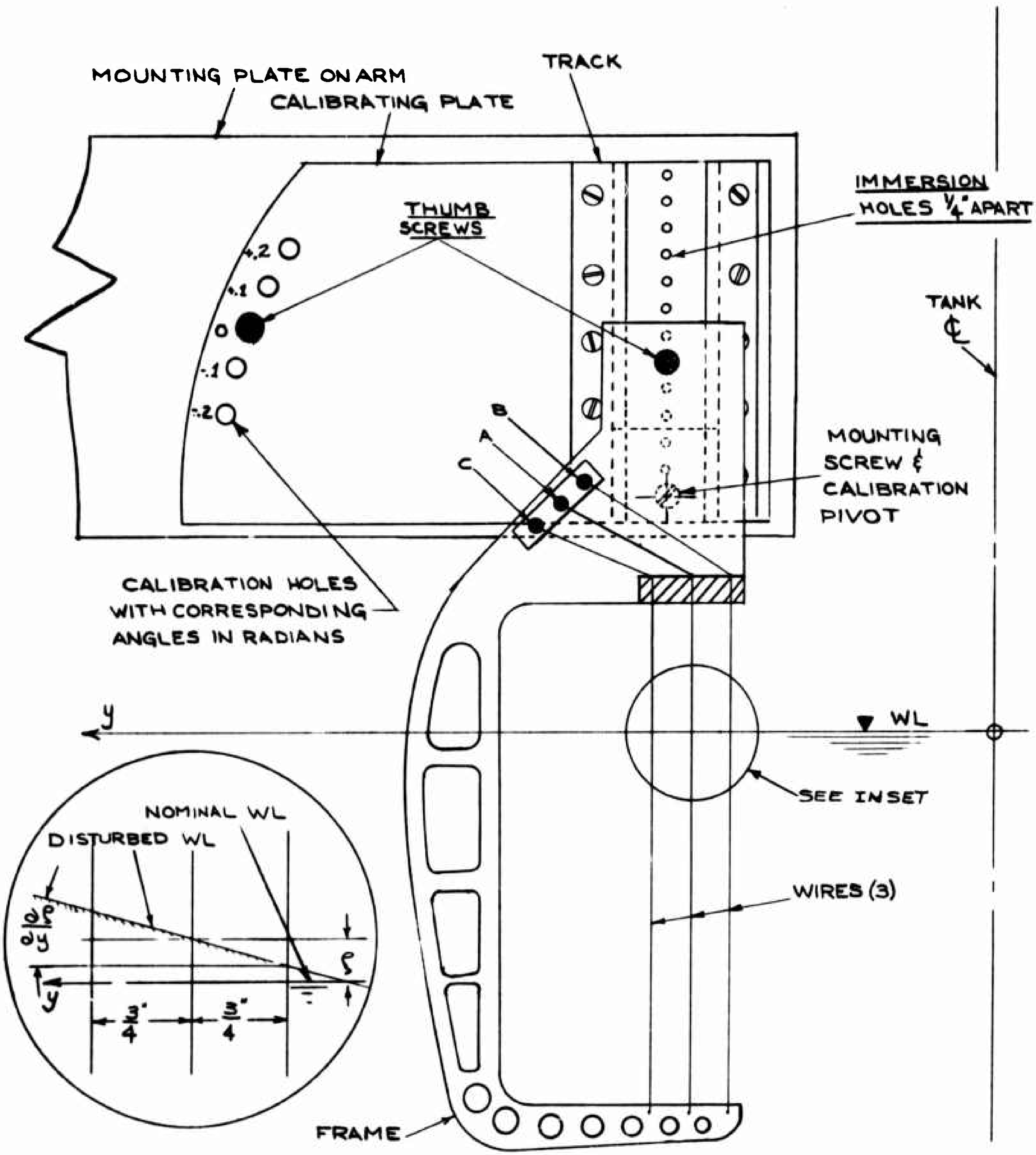

Figure A-1 : Wave-Slope Probe & Calibrating Plate

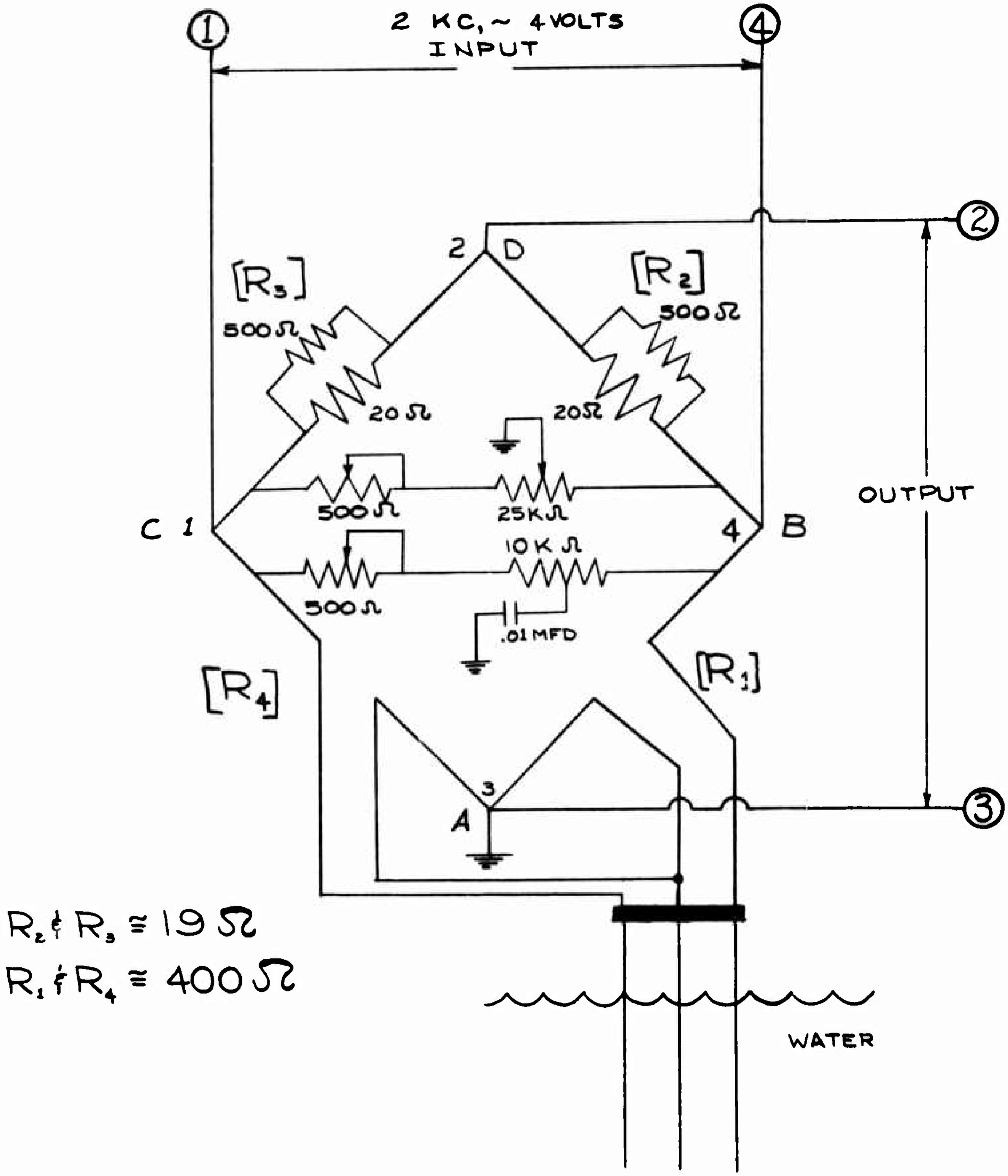

Fig A-2 Wave-Slope Probe Circuit

on the equivalent D.C. resistance of the water elements. In the case of the slope probe the elements are used differentially, as shown. This circuit is more symmetric and actually easier to balance than the height-probe circuit where an electrical resistor must be used to attempt to reproduce the water effect. A combined resistance and capacitance balancing circuit is provided within the bridge circuit to assist that available in the amplifier.

As mentioned previously, good linear response has been established for this probe in operation over the range of $\pm$ 0.20 radians ample for the usual model wave system and relatively good insensitivity with changes in wave height over the range of $\pm$ 1½ inches required for the present tests. Care is taken to keep the wires clean and at constant tension. No difficulty with this arrangement has been noted due to variations in surface film, nor does the exact spacing of the wires seem to be a critical factor as has been noted by others[11]. This might be a fortuitous result of the relatively high exciting frequency of 2KC provided by the Brush amplifier.

The foregoing statements refer to the characteristics noted under static conditions and do not include any effects due to dynamic action, such as meniscus oscillation at the frequency of the waves being measured. Although Pearlman[13] ran a series of such tests on a similar device and found no important effects at the frequencies of importance, the present device should be subjected to a similar series of tests for the sake of completeness.

APPENDIX B: COMPUTER PROGRAM

The computer program for evaluation of the wave resistance and corresponding wave spectrum from the lateral wave-slope record was originally written for an IBM-1130 computer and has since been modified to run on a GE 235 time-sharing computer. This program is shown in Fig. B-1, and is in the FORTRAN language. The basic equations to be computerized are (18) and (19).

The data is composed of approximately 250 wave-slope values (ZETAY) which are read from the recorder tape (see Fig. 3) into storage, along with values for waterline length (LWL), ship speed (VEL), the time increment between data points (DELT), wetted surface (WETSUR), and the number of data points (NOZETA). The program first calculates and stores values of W(NU) for fifty-one values of U(NU) spaced equally from 0 to 5 in steps of 0.10. The latter is controlled by choice of the parameter $k_o b$ (CAYOB) which is related by Equation (23) to an effective tank width b_{eff} (WID). The following computations are then made:

$$\left.\begin{aligned} \mathrm{SY(NU)} &= \sum_{\mathrm{J=1}}^{\mathrm{NOZETA}} \mathrm{ZETAY(J)*SIN(W(NU)*J*DELX)} \\ \mathrm{CY(NU)} &= \sum_{\mathrm{J=1}}^{\mathrm{NOZETA}} \mathrm{ZETAY(J)*COS(W(NU)*J*DELX)} \end{aligned}\right\} \qquad \text{(B-1)}$$

where $\mathrm{DELX = DELT*VEL}$

Successive values of the sines and cosines are computed using trigonometric identities, instead of always depending on the subroutines, to save computational time.

```
1000 $FILE TEST1,TEST2,TEST3
1100 DIMENSION U(100),W(100)
1200 READ,CAYOB,CALIB
1300 DIMENSION ZETAY(250)
1400 READ,DELT,NOZETA
1500 PRINT"WHICH SHIP TEST"
1600 10:INPUT,M
1700 READ(M) VEL,LWL,WETSUR
1800 READ(M) ZETAY
1900 CALIB=85.0
2000 CAYO=32.16/VEL**2
2100 WID=CAYOB/CAYO
2200 FN=VEL/SQRT(32.2*LWL)
2300 VK=VEL*3600./6080.
2400 PRINT"SHIP SPEED IS  ",VK,"  KNOTS,"
2500 PRINT"FROUDE NO. IS",FN
2600 PRINT"WIDTH IS",WID
2700 NONU=0
2800 DO 100 N=1,51
2900  NONU=NONU+1
3000 XNU=N-1
3100 U(N)=3.14159*XNU/CAYOB
3200 W(N)=(.5+(.25+U(N)**2)**.5)**.5
3300 100:CONTINUE
3400 RW=0.
3500 DELX=DELT*VEL
3600 CON=2000.*DELX**2/(WETSUR*CAYOB*CALIB**2)
3700 PRINT"CON IS",CON
3800 PRINT,""NU W(NU) U(NU)  AMP  THETA PHASE WEIGHT     RESISTANCE"
3900 DO 400 NU=1,NONU
4000 SY=0.;CY=0.
4100 SINDEL=SIN(DELX*W(NU)*CAYO)
4200 COSDEL=COS(DELX*W(NU)*CAYO)
4300 SINX=0.
4400 COSX=1.
4500 DO 300 J=1,NOZETA
4600 CY=CY+ZETAY(J)*COSX
4700 SY=SY+ZETAY(J)*SINX
4800 COSINE=COSX*COSDEL-SINX*SINDEL
4900 SINE=SINX*COSDEL+COSX*SINDEL
5000 COSX=COSINE
5100 300:SINX=SINE
5200 THETA=180.*ATAN(U(NU)/W(NU))/3.14159
5300 PHASE=180.*ATAN(SY/CY)/3.14159
5400 AY=SY*SY+CY*CY
5500 AMP=SQRT(AY)
5600 NUU=NU-1
5700 WEIGHT=1./((W(NU)**4)*(2.*W(NU)**2-1.))
5800 R=CON*AY*WEIGHT
5900 RW=RW+R
6000 PRINT 1,NUU,W(NU),U(NU),AMP,THETA,PHASE,WEIGHT,RW
6100 1:FORMAT(I2,2F6.3,3F6.2,F6.4,F15.7)
6200 400:CONTINUE
6300 PRINT"SHIP RESISTANCE IS",RW
6400 GO TO 10
6500 STOP
```

Figure B-1: Computer Program

CY(NU) and SY(NU) are the components of the spectrum which is printed out in terms of the magnitude (AMP) and the phase. These results are used to calculate the wave resistance:

$$C_W = \frac{2\,(DELX)^2}{WETSUR * CAYOB} \sum_{NU=0}^{NONU} \frac{[SY(NU)]^2 + [CY(NU)]^2}{(W(NU))^4 * [2(W(NU))^2 - 1]} \qquad \text{(B-2)}$$

These are then printed out as individual and cumulative values for each NU.

The total running time for this program on the GE-235 is 23 seconds per resistance point plus 13 seconds for compiling or a total of 36 seconds.

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13. ABSTRACT

A new method of determining ship model wave resistance from the wave pattern generated during a test in a model tank is outlined. This method is based on a Fourier analysis of lateral wave-slope data taken on a longitudinal cut paralled to the model path. Results of an exploratory test series carried out at Webb Institute are given and compared with previous results using a different method. The comparison is very encouraging and indicates that a new technique has been established which has many beneficial features, including potential application in full-scale research. Details of an electrical wave-slope measuring probe for use in the model tank are included in an appendix.

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